Applications of delay (delay) (delay) (delay) (delay)

A jaunt through some DSP, applied to electronic music

Rhuaidi Burke

University of Queensland

`rhuaidi.burke@uq.edu.au`

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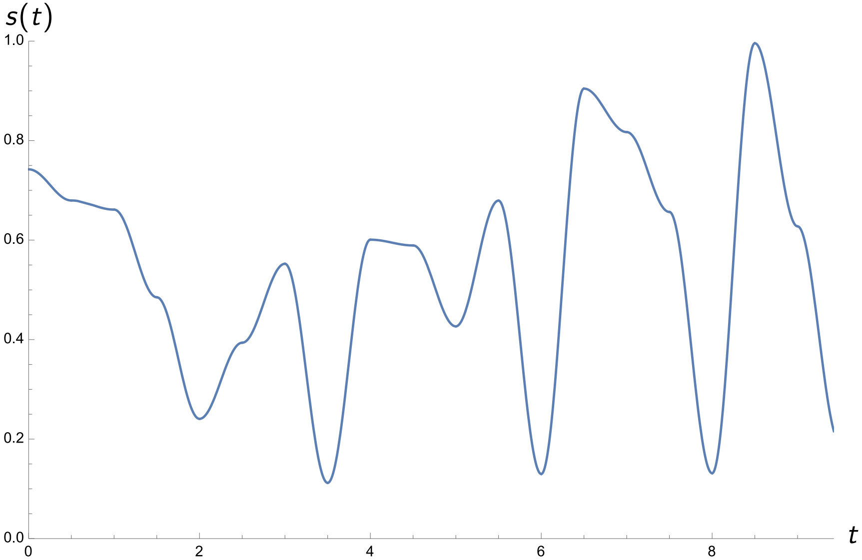
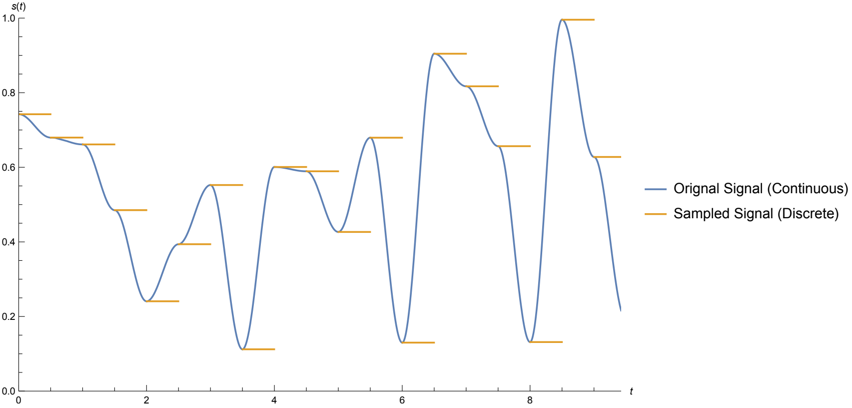
Signals and Samples



Signals and Samples



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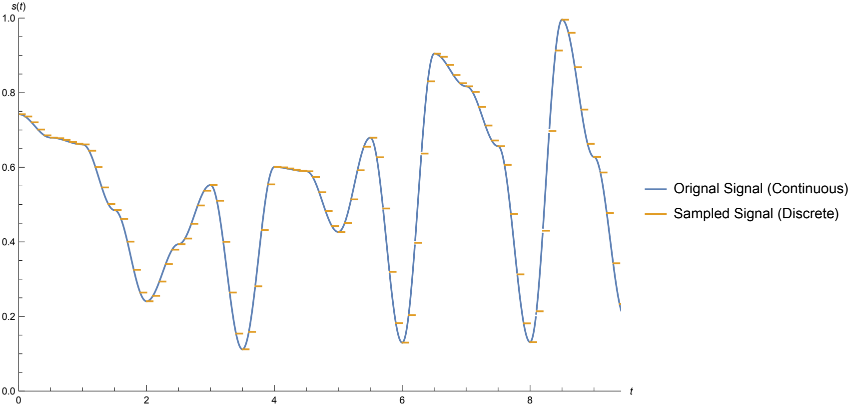


- Typical assumption: our signal has the form of a complex exponential

$$x[n] = z^n, \quad z = e^s, \quad s = \sigma + i\omega.$$

- In general, suppose

$$x[n] = X(z)z^n, \quad y[n] = Y(z)z^n,$$

  where $x[n]$ is some input signal, and $y[n]$ is the output.
  We are often interested in analysing the *transfer function* $H(z)$, of the system defined by

$$H(z) := \frac{Y(z)}{X(z)}.$$

Given $z^n = e^{sn}$, where $s = \sigma + i\omega$, the *frequency response* of the system is given by evaluating $H(i\omega)$.

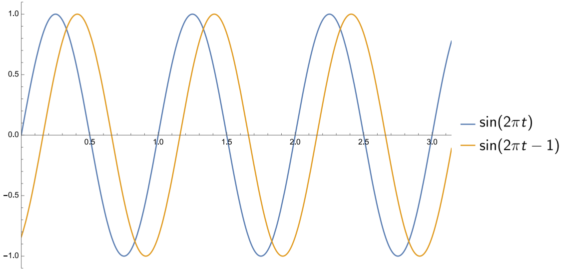
The frequency response describes how the system alters the magnitude and the phase of the input signal frequencies.

Delay

In addition to standard operations (addition, scalar multiplication), the sequence $x[n]$ can be delayed in time by some multiple of the sampling period,

$$y[n] = x[n - M].$$

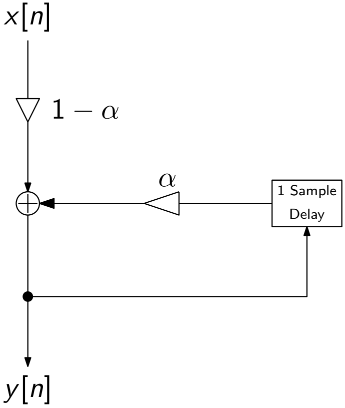
n	1	2	3	4	5	6	7	8	9	10	11	$\cdots$
$x[n]$	0.3	0.6	0.5	0.2	−0.1	−0.4	−0.2	0.2	0.7	0.9	0.8	$\cdots$
$x[n-2]$			0.3	0.6	0.5	0.2	−0.1	−0.4	−0.2	0.2	0.7	0.9



Example: Simple Filter

Consider the input-output system defined by the equation

$$y[n] = (1-\alpha)x[n] + \alpha y[n-1], \quad 0 \leq \alpha \leq 1.$$



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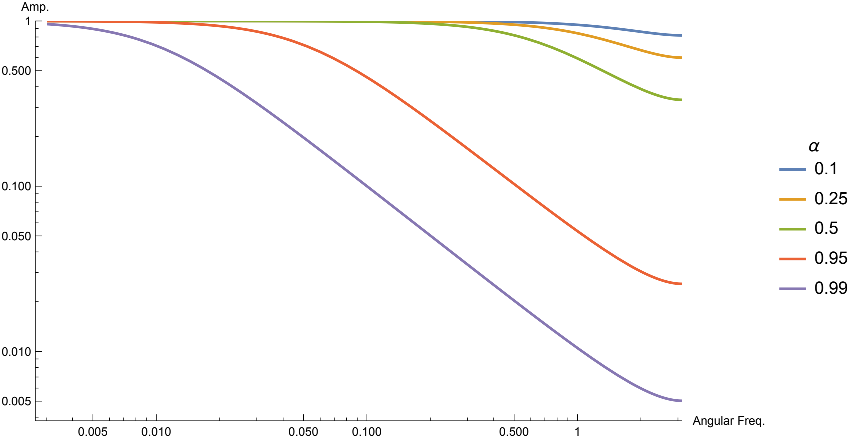
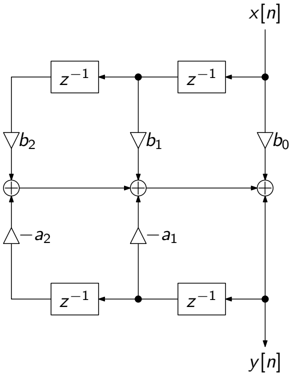
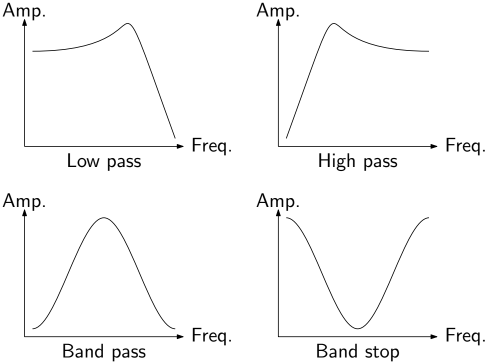


Figure: $|H(i\omega)|$ as a function of ω (log-log scale).

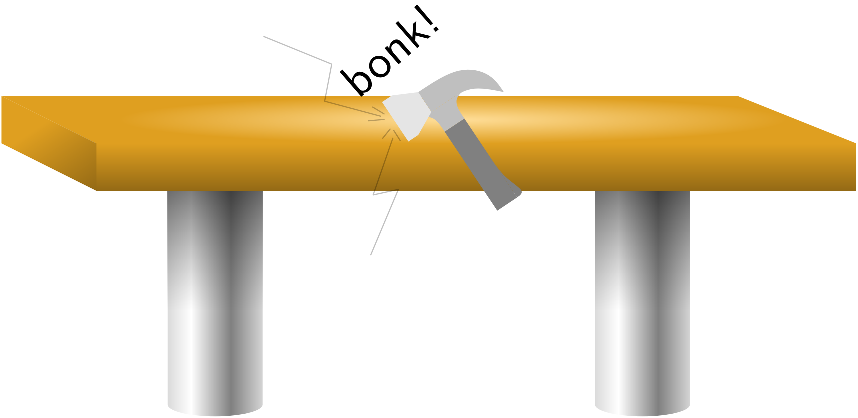
More Filters

By adding more delay lines, more coefficients, and mixing things in various ways, we can cook up all sorts of filters.





Application: Modal Synthesis



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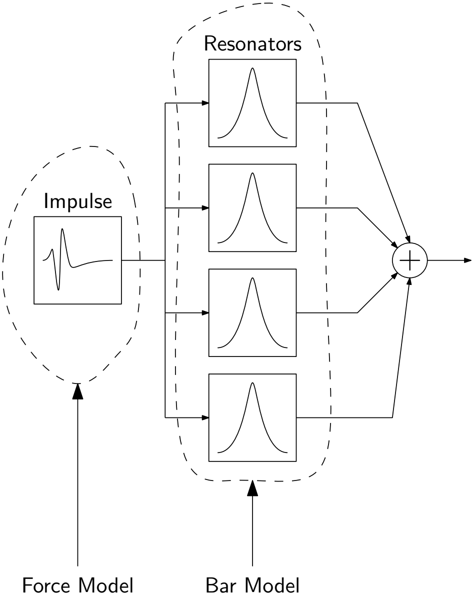
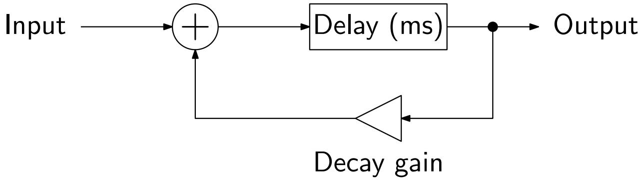


Example: Echo

- Instead of operating on individual samples delay entire “blocks” of the signal (e.g. 1000 samples at a time).
- Produces an *audible* delay.
- Feeding this delayed signal back in on itself, we create a repeating *echo* sound effect.



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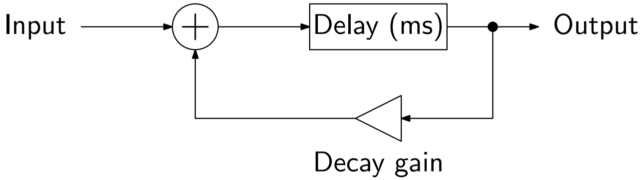


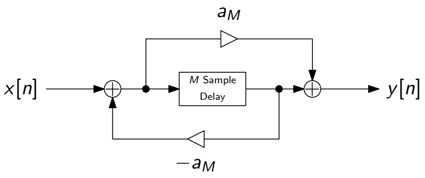
- Modulating the delay time can be used to create interesting audio effects like flangers and phasers.
- We can even use this to model (synthesise) things like plucked or bowed strings.

Example: Reverb

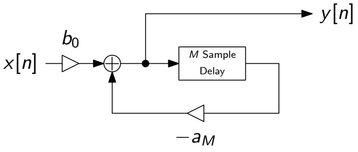
- Reverb creates a sense of *space* by creating a diffuse long-lasting sound from a short input.
- Often created when a sound is reflected on surfaces, causing multiple reflections that build up and then decay as the sound is absorbed by the surfaces of objects in the space.
- Arguably one of the oldest sound effects used in music.
- It is an interesting problem trying to algorithmically model reverb.

Example: Reverb

- We want something which will diffuse the input signal, and has long-lasting, complicated, decay.
- A first approximation to this uses two different kinds of filter: an *all-pass* filter, and a *comb* filter.
- All-pass: no effect on the frequency of the input, but changes the *phase*.
- Comb filters: create constructive and destructive interference.

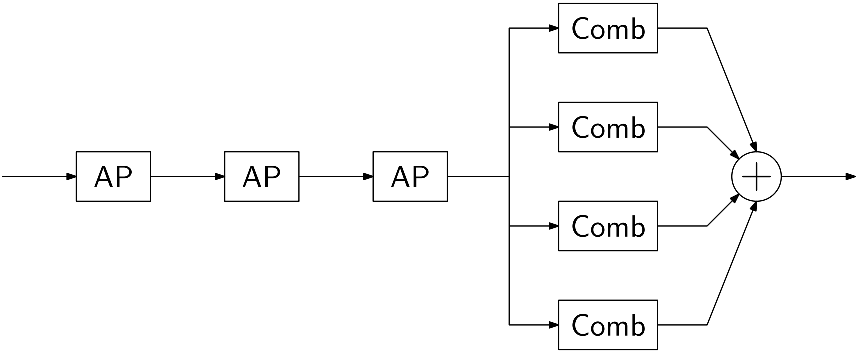


Allpass filter

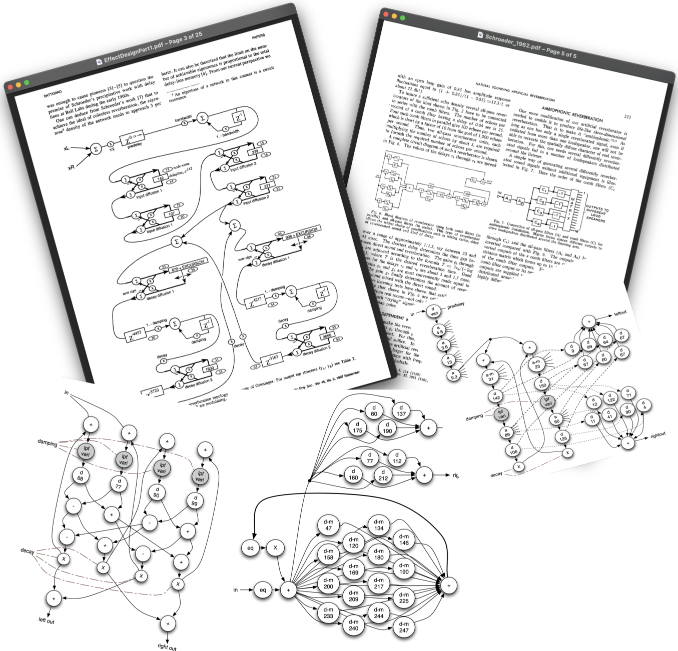


Feedback comb filter

Example: Reverb



Algorithmic reverb design gets crazy!



Some Final Remarks

Many concepts going on behind these ideas, including but not limited to:

- Fourier analysis
- Laplace transforms and $\mathscr{Z}$-transforms
- Difference equations (discrete ODEs)
- Hadamard matrices

Now, a demonstration of reverb + one of my favourite delay effects...