SAA with Benders Cut for a maximal return

FUN FUN TALKS

boomer3boom

September 20, 2025

Problem Context

- Suppose we are a Pokemon card collector interested in a specific pikachu card.
- We want to buy at least 30 of these cards this year and plans to sell them much later in the future.
- Every day we are given the option to buy the card off eBay.
- However, every day the card's value changes, but we have an α level of confidence that Pikachu card will grow by 10% this year.
- How should we buy my pikachu card? (ie At what price should I buy them with what amount should I buy them?)

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- We want a way to capture a range of outcomes in the future since we don't know what will happen.
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My Solution - Stochastic Component

- Monte carlo, Geometric Brownian Motion (GBM), or any variations of GBM.
- I have an alpha value whereby after after the GBM, i pull the price closer to target price: alpha_t * (target_price - next_price).
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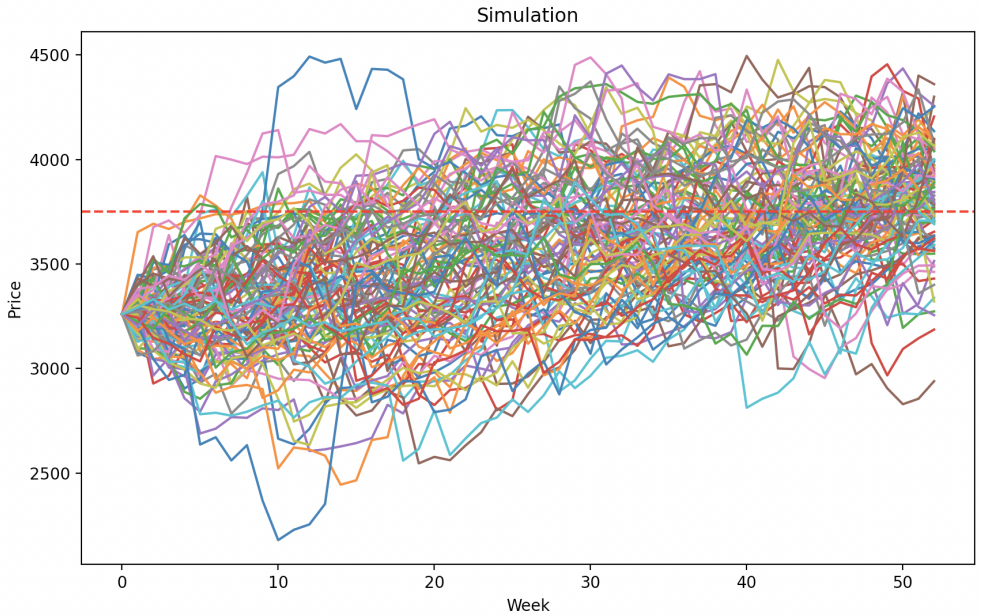
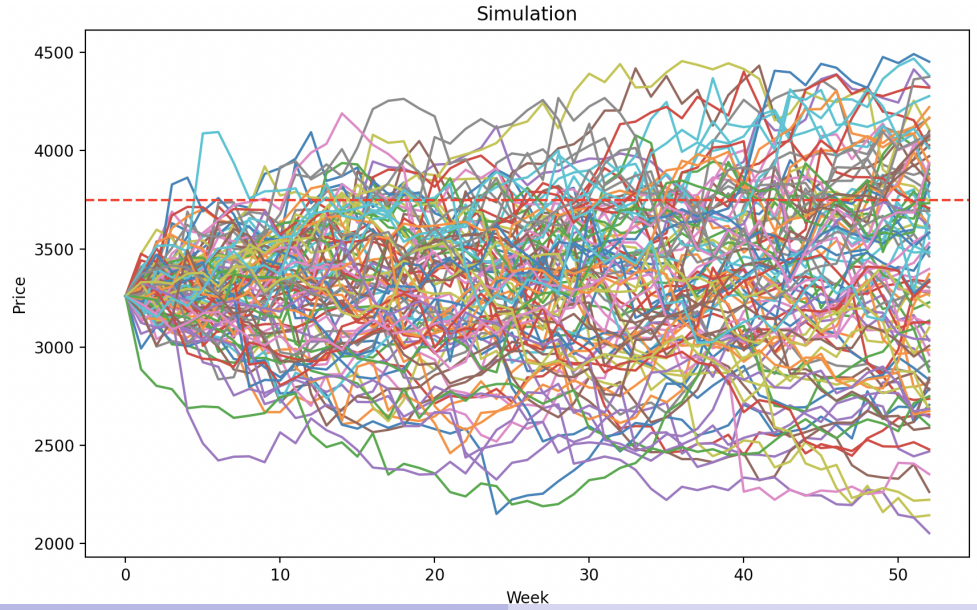
Alpha vs No Alpha





Model?

So. What's our gurobi model?

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- We want to maximize our average return across all scenarios!
- $Z_s \in R$: ROI for scenario s
- $\frac{1}{|S|} \sum_{s \in S} Z_s$

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- At this period, we want to buy an amount of stock at price p, but ... how much have we spent on this simulation?
- At the end of the 52nd week, how much income have generated when we don't exactly know how much was spent and at what price and how many stocks?
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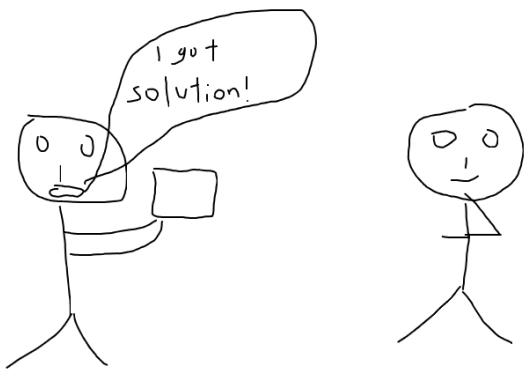

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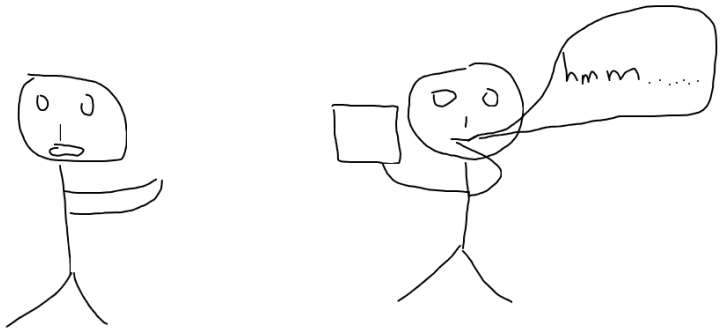

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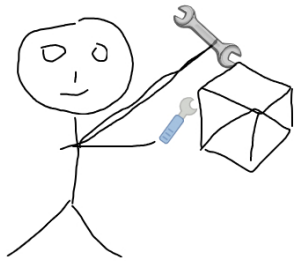

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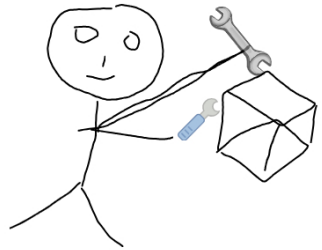

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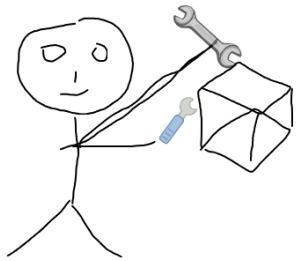

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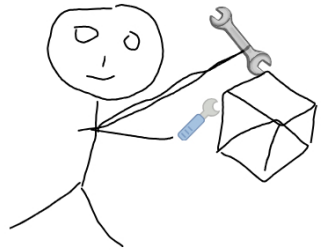

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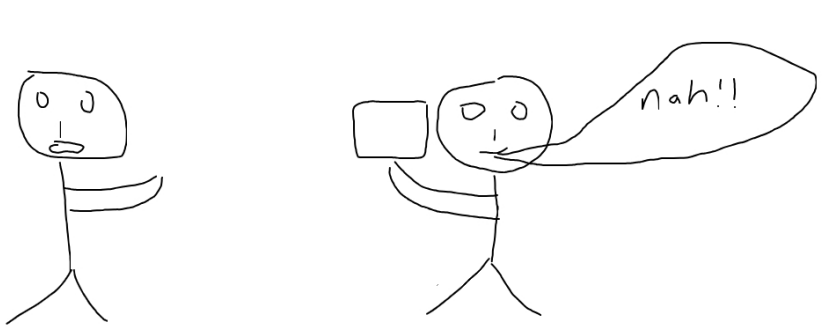

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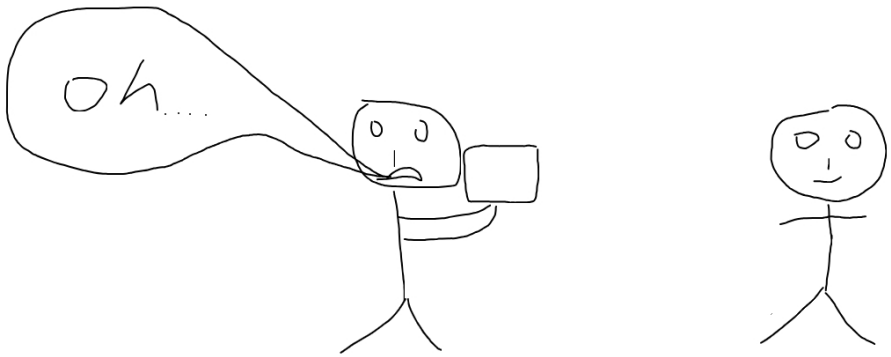

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Bender this Pokemon Card :P ft The Child (Master)

- $Asset_s$: The total value of all your card at the end of the year. The upper bound is Captial/min_price * $Price_{s,52}$.
- $Cost_s$: The cost of buying all your cards at the end of the year. The lower bound is $min(Price_s) * 30$.
- $Z_s \leq Asset_s - Cost_s \quad \forall s \in S$: An initial constraint to prevent the master problem from overshooting.

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- Not a valid solution (feasibility cut) $->$ Provide a constraint that cuts (removes) this solution and (if possible) all the similar solutions that are non-valid.
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- let g(BA, Price) be a function of your choice that takes in the BA result from the child (master problem) and price, list of 52 entries.
- g should output a dictionary that contains 5 keys: cost (true cost), stocks (no of stocks), assets (final asset), invalids (list of violation), and ROI.
- list of violations might include: Need More Stocks, or Spend Too Much.

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How to generate better cuts

BA = {1: 5, 2: 4, 3: 4, 6: 6, 7: 0}

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- BA = {1: 5, 2: 4, 3: 4, 6: 3, 7: 0}?
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Spend Over Capital:

- BA = {1: 7, 2: 4, 3: 4, 6: 6, 7: 0}?
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Not Enough Cards:

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Valid Solution but wrong objective

$$Z_s \leq g(BA, prices)[ROI] + (500000 * \sum_{(p,a)inCurrent} 1 - BA_{p,a})$$