Yes, It Can (Theoretically) Run Doom — Magic: The Gathering is Turing Complete

Max Orchard

October 27, 2023

Disclaimer

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- While this talk would probably suit UQCS better, I claim it's a Maths Talk because MATH3306 (which I have never taken) discusses Turing machines

What is Magic: The Gathering?

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Players build decks of at least 60 cards (out of a total of more than 27000 distinct cards), and play against opponents who have their own unique deck.

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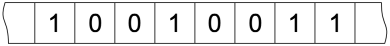
1. A *tape*, consisting of a sequence of discrete *cells*. Each cell contains a symbol from some finite alphabet, and the tape extends indefinitely to the left and right.



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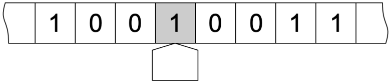
2. A *head* that is positioned over some cell (the *current cell*). It can read a symbol from and write a symbol to that cell.



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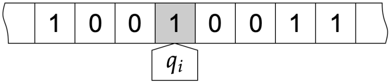
3. A *state register* that stores the current "state" (of finitely many) that the Turing machine is in.



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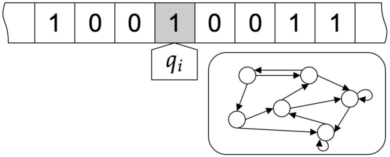
4. A (finite) *state table* that, given the current state and current cell, will cause the Turing machine to do, in sequence (potentially skipping some steps):
   1. write a symbol to the current cell
   2. move the head left or right one cell
   3. change the current state



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Though conceptually simple, a Turing machine is capable of executing any program a real computer can execute. In fact, a Turing machine's indefinite memory makes it very powerful!

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Current Cell		**0**	**1**
Current State A	write	1	1
	move	R	L
	state	B	C
Current State B	write	1	1
	move	L	R
	state	A	B
Current State C	write	1	1
	move	L	N
	state	B	HALT

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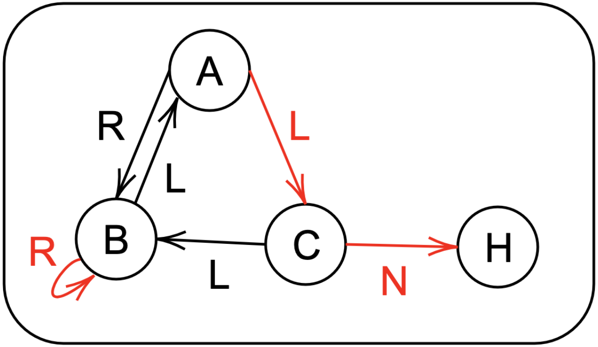

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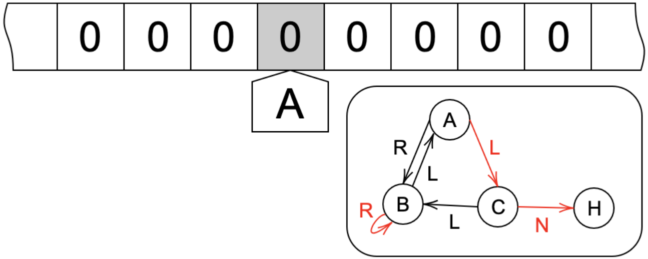

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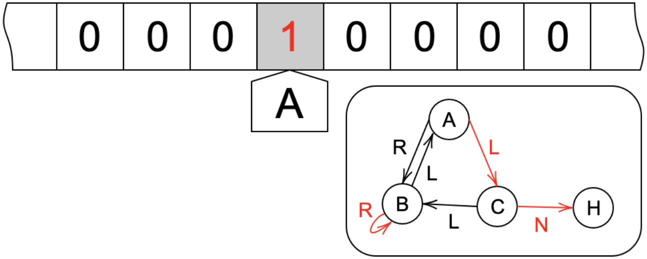

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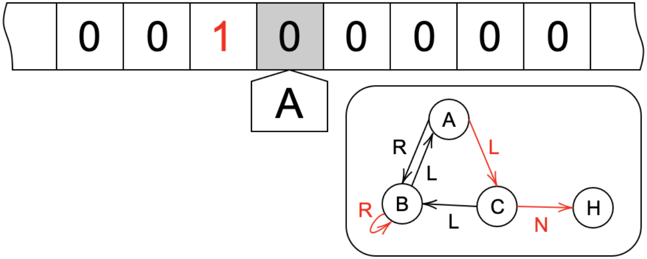

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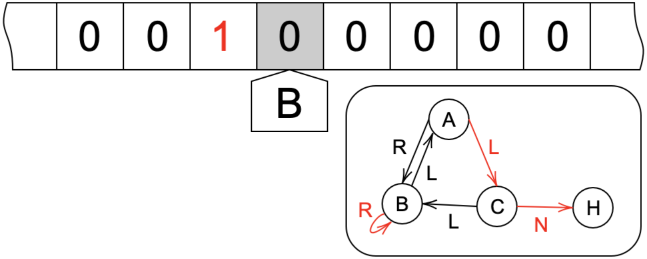

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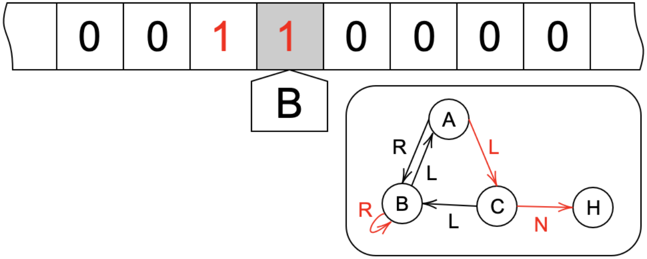

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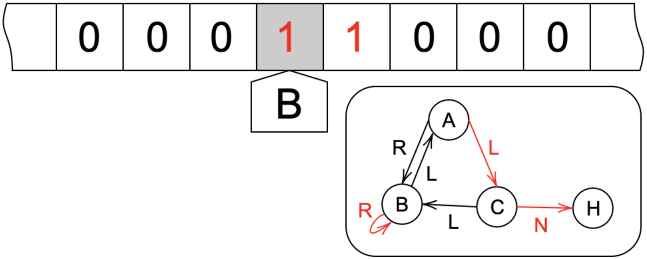

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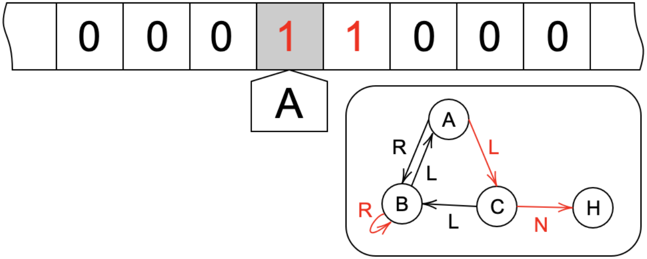

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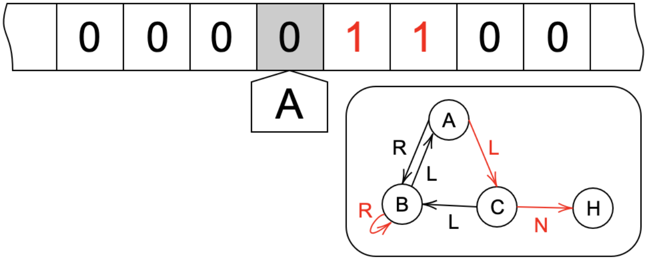

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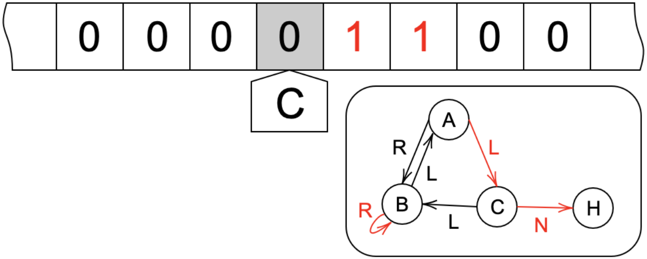

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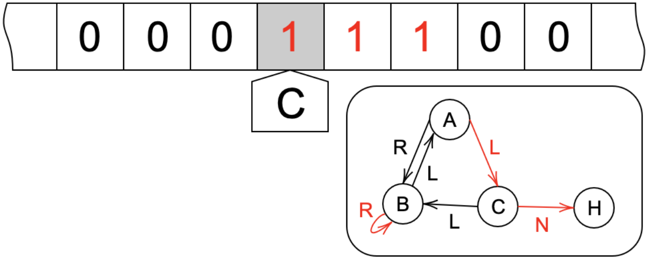

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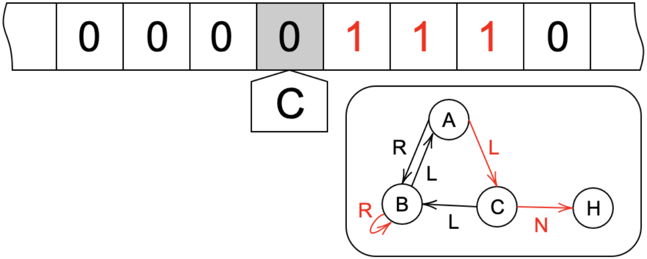

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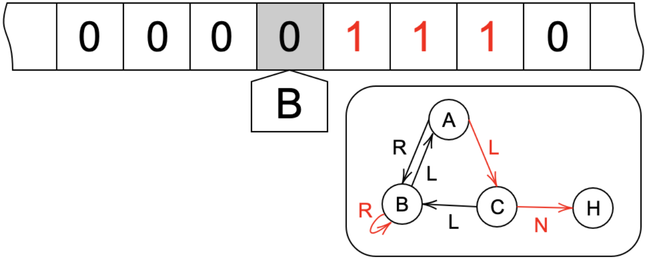

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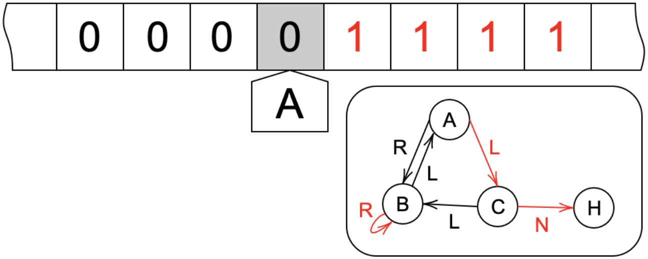

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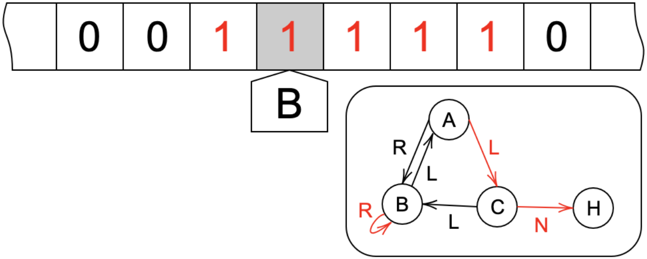

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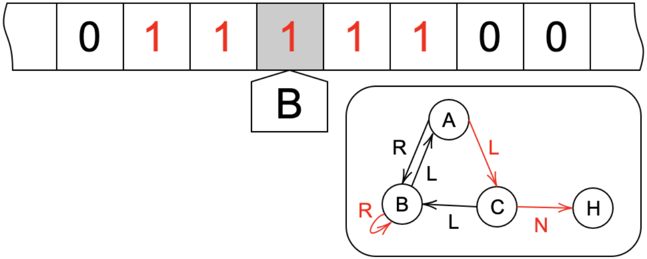

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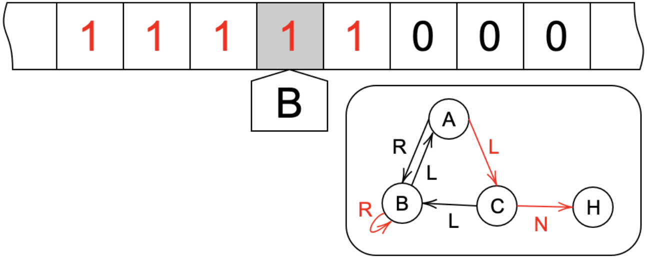

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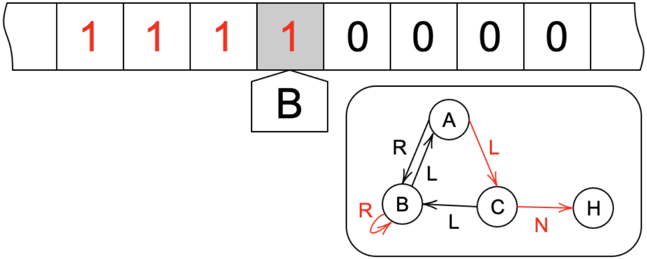

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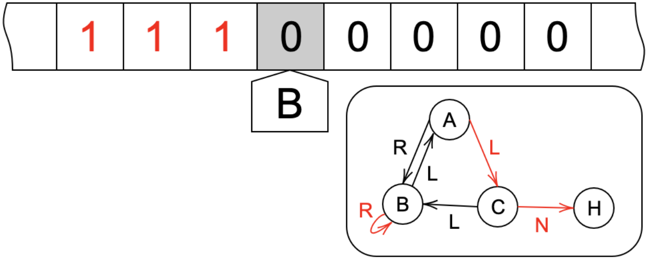

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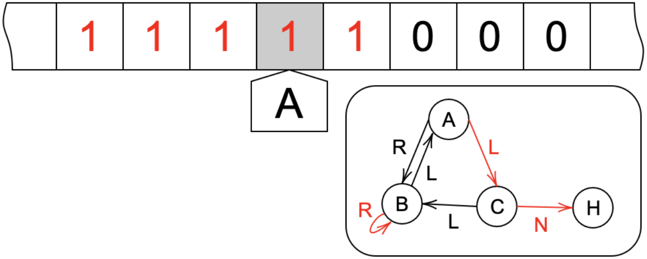

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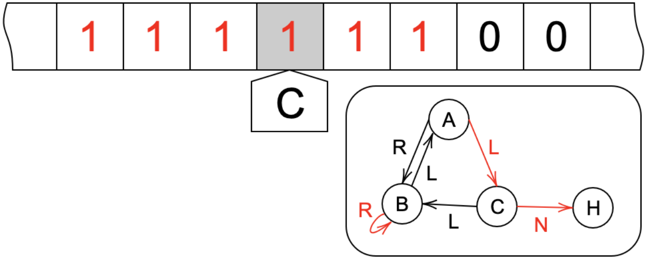

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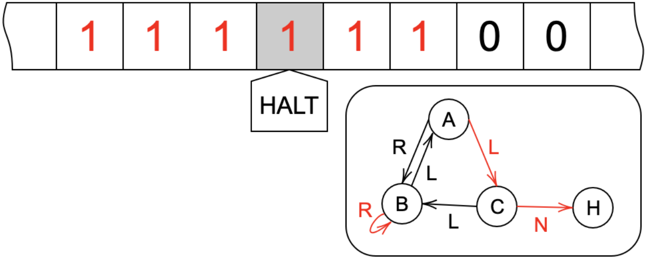

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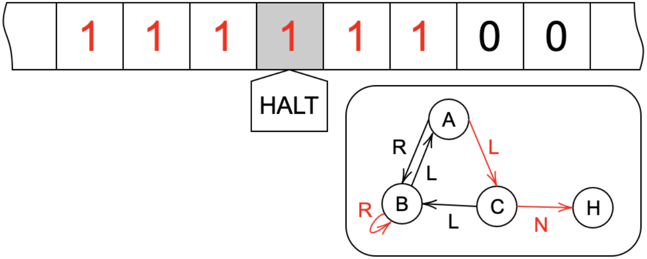
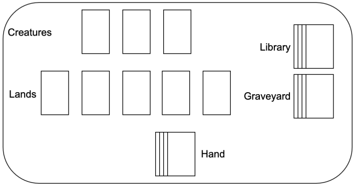


This is known as a 3-state *busy beaver*.

Back to MTG

Each player has a deck of at least 60 cards, which they shuffle at the beginning of the game and place in their *library*. They then draw 7 cards, which form their *hand*.



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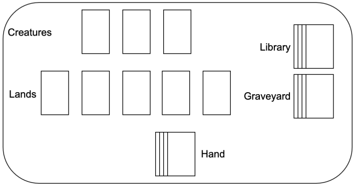

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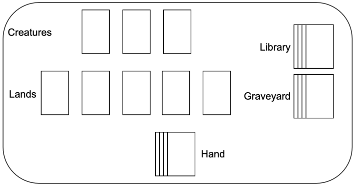
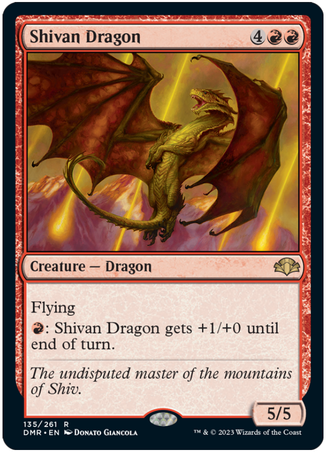
When a card is removed from the field (e.g. a creature dies), it moves to the *graveyard* and is out of play.



Creature Cards

Creatures are *permanents*, that is they stay on the field after they are cast.


Shivan Dragon
4
Creature — Dragon
Flying
: Shivan Dragon gets +1/+0 until end of turn.
The undisputed master of the mountains of Shiv.
5/5
135/261 R
DMR • EN Donato Giancola
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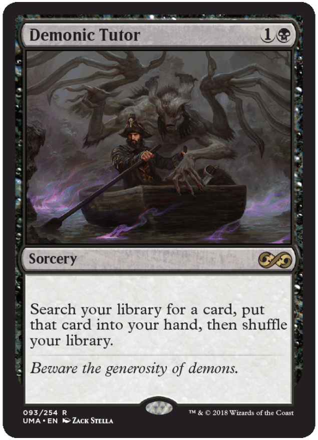
Sorcery Cards

Sorceries are spells that have an immediate effect when cast, and are removed from play once their effect resolves.



Demonic Tutor 1

Sorcery

Search your library for a card, put that card into your hand, then shuffle your library.

Beware the generosity of demons.

093/254 R
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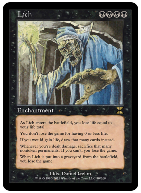
Computability Concerns

The issue of computability immediately arises. Consider the following situation: each player controls **Lich**, **Transcendence**, and **Laboratory Maniac**.



Lich

Enchantment

As Lich enters the battlefield, you lose life equal to your life total.

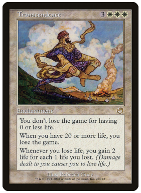
You don't lose the game for having 0 or less life.

If you would gain life, draw that many cards instead.

Whenever you're dealt damage, sacrifice that many nontoken permanents. If you can't, you lose the game.

When Lich is put into a graveyard from the battlefield, you lose the game.

Illus. Daniel Gelon



Transcendence

Enchantment

You don't lose the game for having 0 or less life.

When you have 20 or more life, you lose the game.

Whenever you lose life, you gain 2 life for each 1 life you lost. *(Damage dealt to you causes you to lose life.)*

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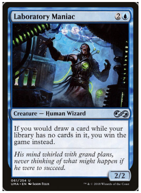

Laboratory Maniac

Creature — Human Wizard

If you would draw a card while your library has no cards in it, you win the game instead.

His mind whirled with grand plans, never thinking of what might happen if he were to succeed.

2/2

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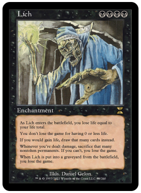
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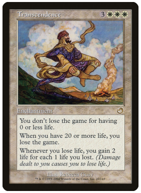
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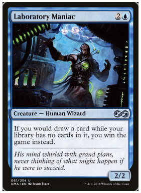
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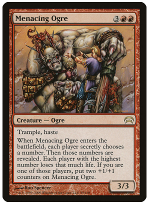
One player then casts **Menacing Ogre**.


Menacing Ogre

Creature — Ogre

Trample, haste

When Menacing Ogre enters the battlefield, each player secretly chooses a number. Then those numbers are revealed. Each player with the highest number loses that much life. If you are one of those players, put two +1/+1 counters on Menacing Ogre.

3/3

Ron Spencer

Forced Moves

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For this reason, the Turing machine will be constructed so that execution happens using forced moves only, so that a player can't decide to stop performing the computation simply because they feel like it.

Constructing the Turing Machine

This game of MTG will be played between Gary and Rupert.

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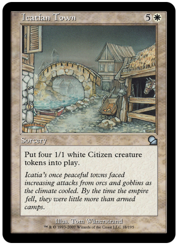
Each component of the Turing machine will be represented by different types of cards or actions that Gary and Rupert control/take.

We will be constructing the “Rogozhin $(2, 18)$ universal Turing machine”, which is able to encode any Turing machine using 2 states and 18 symbols (this is complicated and I don’t fully understand the details, but we’ll go along with it).

Tape

MTG has "token" creatures that can be created from spell effects, and are not represented by any specific card. We will use tokens to represent the cells of the tape, with the creature type representing the symbol.



Icatian Town 5

Sorcery

Put four 1/1 white Citizen creature tokens into play.

Icatia's once peaceful towns faced increasing attacks from orcs and goblins as the climate cooled. By the time the empire fell, they were little more than armed camps.

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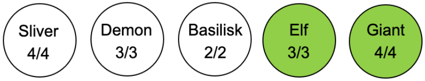
MTG has no concept of adjacency. All creatures in play are simply lumped together into a multiset (or bag). This presents a problem — how do we encode the position of cells with respect to the head?

Tape

We do this by using colours to represent which side of the head a cell is on (white for left, green for right), and power/toughness to represent distance from the head.



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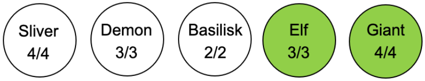
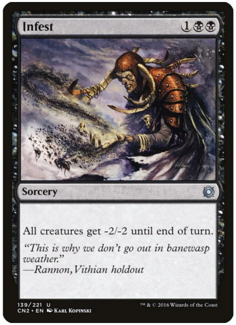


These tokens will be controlled by Rupert, except for the most recently created token which will be controlled by Gary.

Head

Reading the current cell will be done by Gary casting **Infest**.



All creatures get -2/-2 until end of turn.

"This is why we don't go out in banewasp weather."
—Rannon, Vithian holdout

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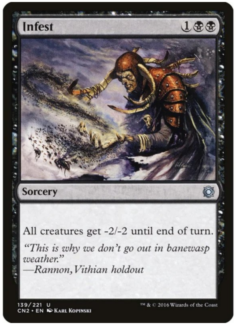

Infest

1

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—Rannon, Vithian holdout

139/221 U
CN2 • EN Karl Kopinski

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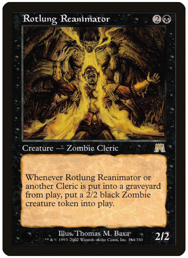
This will kill the unique 2/2 token (the current cell), "reading" it. The state register will detect this, and create a new 2/2 token, "writing" a new symbol to the cell.

State Table

The state table will be encoded using **Rotlung Reanimator** and **Xathrid Necromancer** cards.



Rotlung Reanimator 2

Creature — Zombie Cleric

Whenever Rotlung Reanimator or another Cleric is put into a graveyard from play, put a 2/2 black Zombie creature token into play.

Illus. Thomas M. Baxa 2/2

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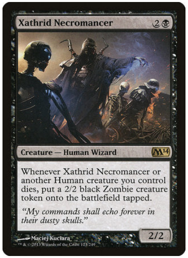

Xathrid Necromancer 2

Creature — Human Wizard

Whenever Xathrid Necromancer or another Human creature you control dies, put a 2/2 black Zombie creature token onto the battlefield tapped.

"My commands shall echo forever in their dusty skulls."

Maciej Kuciara 2/2

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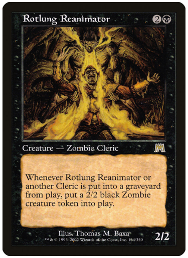
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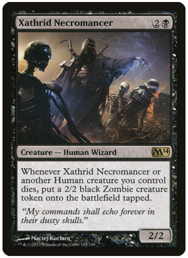
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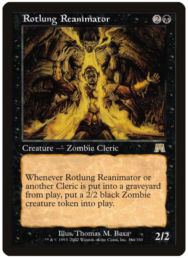

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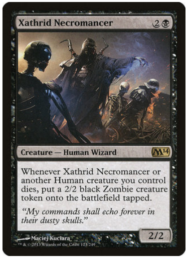

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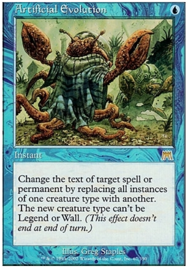
These cards detect when a specific creature type dies, and creates a new token.

Rotlung Reanimator will be used to keep the current state, and **Xathrid Necromancer** to change state (i.e. the "tapped" state of the tokens).

State Table

We can edit the card text of **Rotlung Reanimator** and **Xathrid Necromancer** using **Artificial Evolution** and **Glamerdye**.


Artificial Evolution

Instant

Change the text of target spell or permanent by replacing all instances of one creature type with another. The new creature type can't be Legend or Wall. *(This effect doesn't end at end of turn.)*

Illus. Greg Staples

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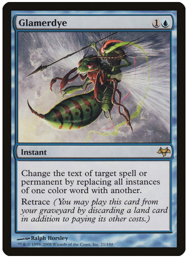
Glamerdye 1

Instant

Change the text of target spell or permanent by replacing all instances of one color word with another.

Retrace *(You may play this card from your graveyard by discarding a land card in addition to paying its other costs.)*

Ralph Horsley

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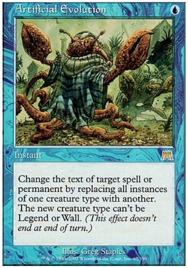

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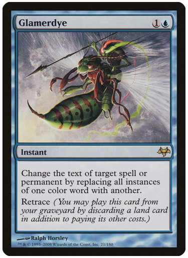

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This effectively means we have cards that say "If `<creature type 1>` dies, create a 2/2 `<colour>` `<creature type 2>`", which encode both the write and move steps in the state table.

State Register

To change state, we use a game mechanic called *phasing*.

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Phasing cards that a player controls alternate between “phased in” and “phased out” at the start of their turn. A phased out card is treated as though it doesn’t exist. For this reason, a computation step will take 4 turns if we are not changing states, and 3 turns otherwise.

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We grant phasing to creatures by using **Cloak of Invisibility**.

Cloak of Invisibility
Enchant Creature
Enchanted creature gains phasing and cannot be blocked except by Walls.
"Useful . . . if you can find it."
—Pashad ibn Asim, Suq'Ata trader
Illus. John Coulthart

Computational Step Outline

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5. Rupert does nothing

Initial State

At the start of a computation step, Gary has one card in his hand, **Infest**. His library consists of **Cleansing Beam**, **Coalition Victory**, and **Soul Snuffers** in order.

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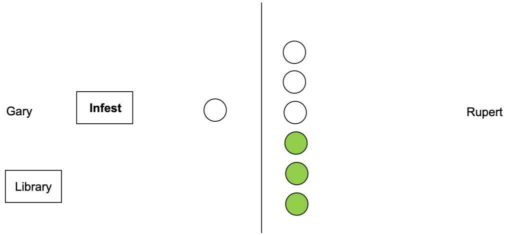
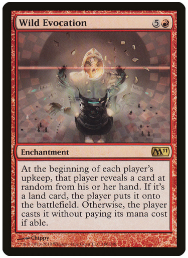


Casting Spells

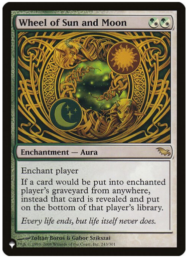
Rupert controls **Wild Evocation**, which forces Gary to play the only card in his hand.

Gary controls **Wheel of Sun and Moon**, which causes these cards to be recycled into his library. After Gary plays his card, he draws the next one in his library.


Wild Evocation 5

Enchantment

At the beginning of each player's upkeep, that player reveals a card at random from his or her hand. If it's a land card, the player puts it onto the battlefield. Otherwise, the player casts it without paying its mana cost if able.


Wheel of Sun and Moon

Enchantment — Aura

Enchant player

If a card would be put into enchanted player's graveyard from anywhere, instead that card is revealed and put on the bottom of that player's library.

Every life ends, but life itself never does.

There are cards in play preventing players from taking any other action.

Turn 1

Gary is forced to cast **Infest**, performing the read/write step. The new token's colour represents the direction the tape will move — white for left and green for right.

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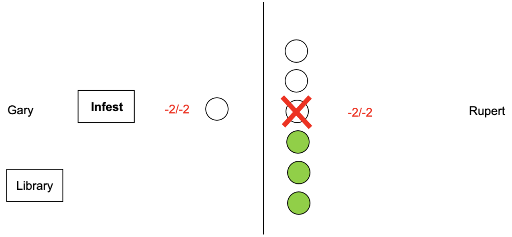

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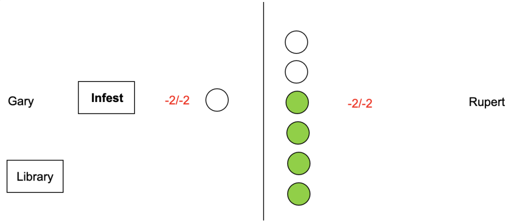

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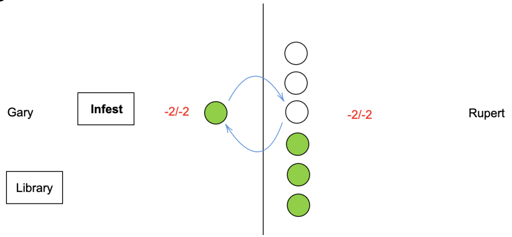
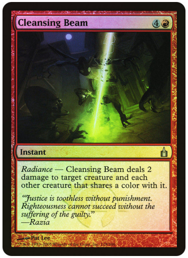


Gary controls **Illusory Gains**, which causes Gary to take control of the current cell and Rupert to take control of Gary's previous token.

Turn 2

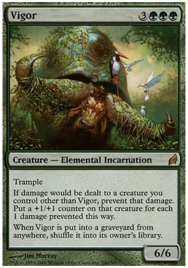
Gary is forced to cast **Cleansing Beam**. There are cards in play to force Gary to target his own token (the current cell). Both players control **Vigor**, which instead *adds* power/toughness whenever damage is dealt.


Cleansing Beam 4

Instant

Radiance — Cleansing Beam deals 2 damage to target creature and each other creature that shares a color with it.

"Justice is toothless without punishment. Righteousness cannot succeed without the suffering of the guilty."
—Razia


Vigor 3

Creature — Elemental Incarnation

Trample

If damage would be dealt to a creature you control other than Vigor, prevent that damage. Put a +1/+1 counter on that creature for each 1 damage prevented this way.

When Vigor is put into a graveyard from anywhere, shuffle it into its owner's library.

6/6

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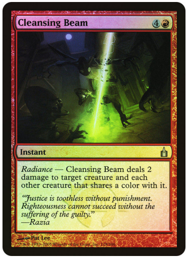

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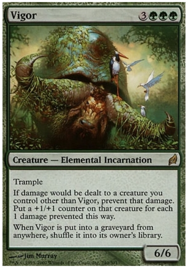

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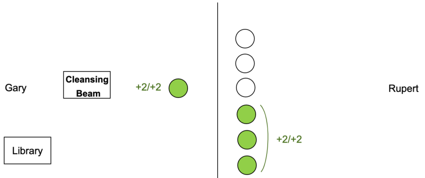

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If the state table determines that we are changing states, the token that is sent to Gary in Turn 1 is *tapped*. At the beginning of each turn, all tapped cards are *untapped*.

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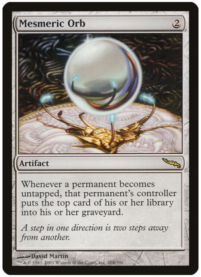
Mesmeric Orb 2

Artifact

Whenever a permanent becomes untapped, that permanent's controller puts the top card of his or her library into his or her graveyard.

A step in one direction is two steps away from another.

David Martin

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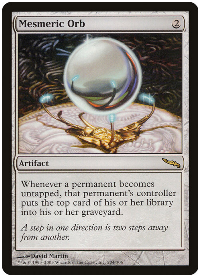
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Mesmeric Orb causes **Coalition Victory** to be skipped! This shortens the number of turns in a computational step by one, changing the state.

Turns 3 and 4

If **Mesmeric Orb** doesn't trigger, Turn 3 involves Gary being forced to cast **Coalition Victory**, which does nothing*.

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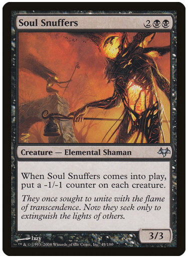
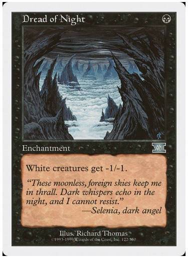

Soul Snuffers
Creature — Elemental Shaman
When Soul Snuffers comes into play, put a -1/-1 counter on each creature.
They once sought to unite with the flame of transcendence. Now they seek only to extinguish the lights of others.
3/3


Dread of Night
Enchantment
White creatures get -1/-1.
"These moonless, foreign skies keep me in thrall. Dark whispers echo in the night, and I cannot resist."
—Selenia, dark angel
Illus. Richard Thomas

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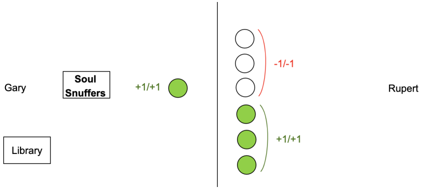

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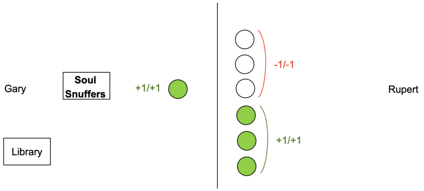

We have completed one computational step. **Infest** is now in Gary's hand, and we are ready to start the next step.

Halting

Halting in this $(2, 18)$ UTM is given by reading a certain symbol in a certain state. This is encoded by the state table (i.e. **Rotlung Reanimator**), and will result in a token being created with a specific colour and type (blue Assassin).

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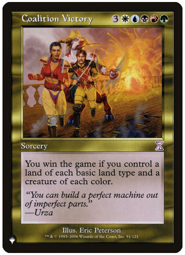
Halting in this $(2, 18)$ UTM is given by reading a certain symbol in a certain state. This is encoded by the state table (i.e. **Rotlung Reanimator**), and will result in a token being created with a specific colour and type (blue Assassin).

Coalition Victory causes Gary to win the game immediately if he satisfies certain conditions.



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"You can build a perfect machine out of imperfect parts."
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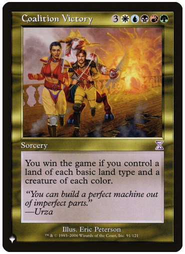

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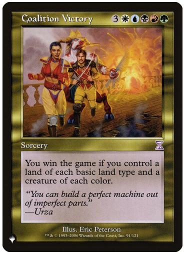

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If the machine does not halt, the game will be in an unbreakable deterministic infinite loop, which is a draw by rule.

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We simply mark the ends of the tape with unused token types, and detect when they are read. A similar process to writing cells normally can be followed to create a new cell (i.e. with a **Rotlung Reanimator**).

Tournament Legal Deck

This is the full tournament-legal deck of cards.

	Card	Purpose		Card	Purpose		Card	Purpose
4	Ancient Tomb	Bootstrap	1	Rotlung Reanimator	Logic processing	1	Xathrid Necromancer	Change state
4	Lotus Petal	Bootstrap	1	Cloak of Invisibility	Logic processing	1	Mesmeric Orb	Change state
4	Grim Monolith	Infinite mana device	1	Infest	Logic processing	1	Coalition Victory	Halting device
4	Power Artifact	Infinite mana device	1	Cleansing Beam	Logic processing	1	Prismatic Omen	Halting device
4	Gemstone Array	Infinite mana device	1	Soul Snuffers	Logic processing	1	Choke	Halting device
4	Staff of Domination	Draw rest of deck	1	Illusory Gains	Logic processing	1	Recycle	Remove choices
1	Memnarch	Make token copies	1	Privileged Position	Logic processing	1	Blazing Archon	Remove choices
1	Stolen Identity	Make token copies	1	Steely Resolve	Logic processing	1	Djinn Illuminatus	Simplify setup
1	Artificial Evolution	Edit cards	1	Vigor	Logic processing	1	Reito Lantern	Simplify setup
1	Olivia Voldaren	Edit cards	1	Fungus Sliver	Logic processing	1	Claws of Gix	Simplify setup
1	Glamerdye	Edit cards	1	Dread of Night	Logic processing	1	Riptide Replicator	Set up tape
1	Prismatic Lace	Edit cards	1	Wild Evocation	Forced play device	1	Capsize	Set up tape
1	Donate	Edit card control	1	Wheel of Sun and Moon	Forced play device	1	Karn Liberated	Cleanup after setup
1	Reality Ripple	Edit card phase	1	Shared Triumph	Infinite tape device	1	Fathom Feeder	Cleanup after setup

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This deck costs about US$2300 (unofficially).

References

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