# Please Not First Past the Post

## An Introduction to Voting Theory

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- It is inevitable that I will bring up examples from politics for discussion. (There are many interesting elections!)
- I will try to avoid political discussions. This is not a politics talk, this is a maths talk! Or at the very least, a talk about elections, not those who are elected.

# Introduction

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- Idea: **simple**.
- In practice: **very difficult** to get right.

Goal of this talk: make you think about elections.[1]

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# Introduction

The MSS election in 20XX for presidency is being contested between two candidates:

- Alice.
- Bob.

Who should be president?

# First-past-the-post

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One vote per voter. After each voter votes, the candidate who wins more than half the votes wins.

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For two candidates...fair enough. Even with people abstaining from voting, one would expect this to give the “fairest” result. Not much analysis needed here. However...

# Multiple candidates

The MSS election in 20XY for presidency is being contested between two candidates:

- Alice.
- Bob.
- Charlie.

Who should be president?

# First-past-the-post (again)

Majority method probably won't result in a winner. The most natural extension is:

**Method 2 (First-Past-The-Post Method, plurality)**

One vote per voter for their most preferred candidate. After each voter votes, the candidate who wins **the most** votes wins.

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Any problems?[2]

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# FPTP by cats

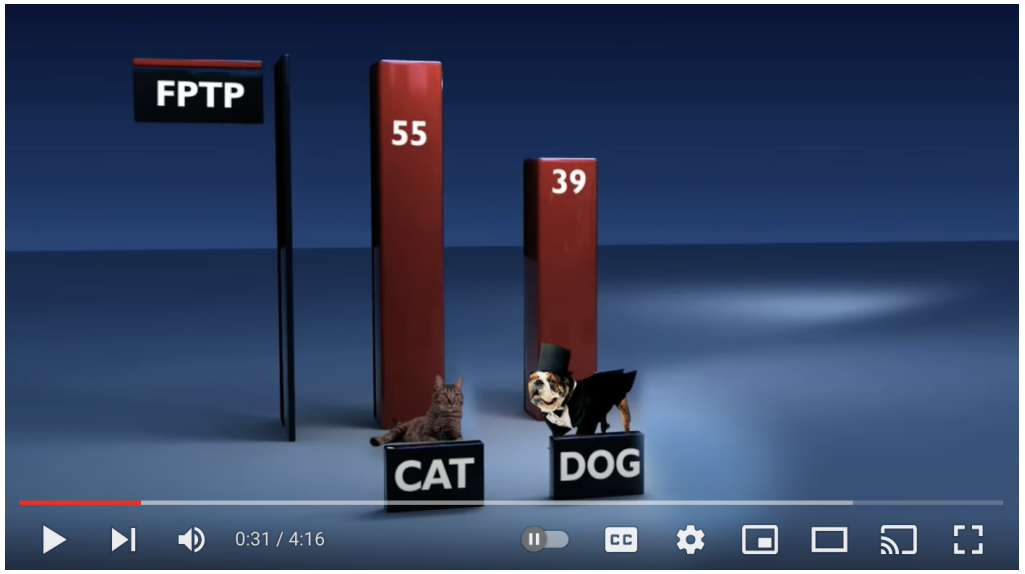


**Figure:** Is your Cat confused about the alternative vote?

# The problems with FPTP

| Party | Candidate | Votes |
|---|---|---|
| Republican | George W. Bush | 2,912,790 |
| Democratic | Al Gore | 2,912,253 |
| Green | Ralph Nader | 97,488 |
| Reform | Patrick Buchanan | 17,484 |

**Figure:** 2000 United States presidential election in Florida. Bush won by 537 votes.

[3] Some say Bush's victory was due to Nader being a spoiler, some say it was due to the "butterfly ballot" controversy. If you're curious, ask me.

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**Figure:** 2000 United States presidential election in Florida. Bush won by 537 votes.

This was arguably a turning point in US politics, caused (in a large part) by a less-than-perfect electoral system. For reasons regarding the Electoral College, these 537 votes decided 25 electors which in turn confirmed the presidency for Bush.[3]

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# The problems with FPTP

- Tactical voting (i.e. “anything but the runner-up is a vote for the winner”).[4]

[4] There was a referendum to introduce preferential voting in 2011 in the UK; major parties campaigned this point almost exclusively.

[5] when the winner of the majority of the popular vote loses the election.

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- Tactical voting (i.e. “anything but the runner-up is a vote for the winner”).[4]
- Wasted votes
- Easier to gerrymander
- Unrepresentative
- Can result in “majority reversal”[5] when used at-scale in electoral systems (see 2016 US presidential election).

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# The problems with FPTP

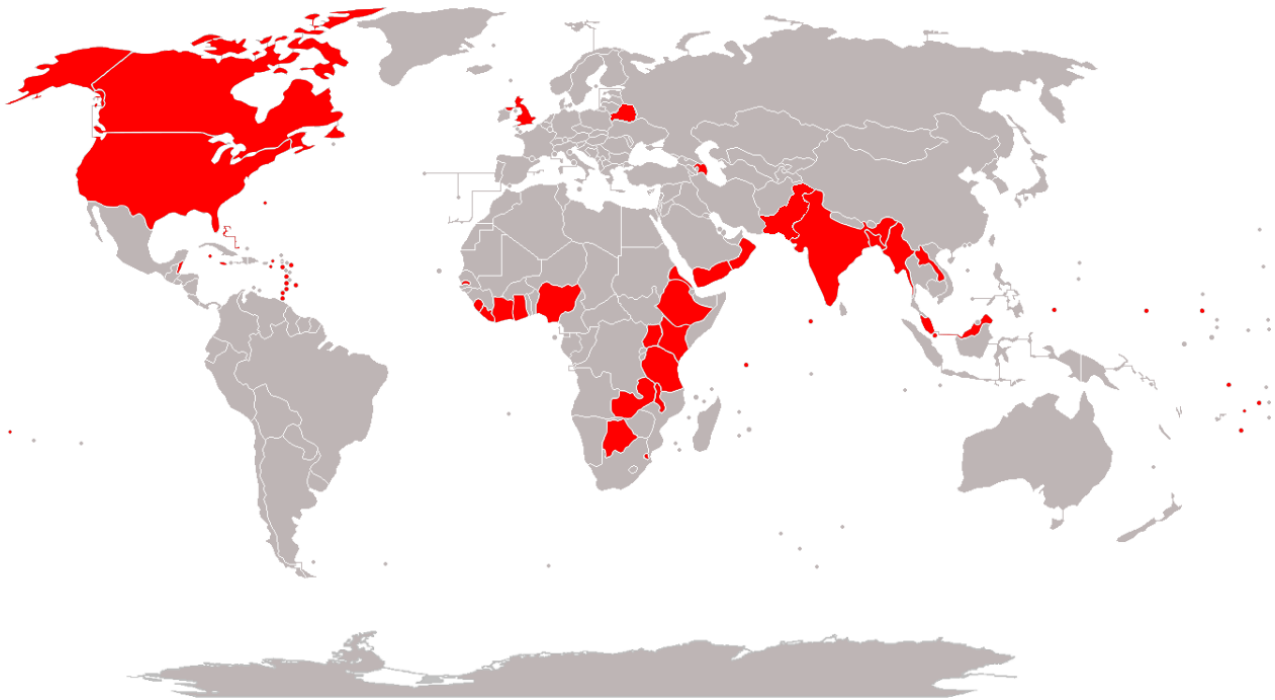

**Figure:** Countries that use first-past-the-post for their national legislative elections.

# What other options are there?

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- Run **multiple elections**, whittle the size down.
- Allow people to **preference** candidates.
- Allow people to allocate some number of **points** to each candidate.

# Runoff elections

**Method 3 (Runoff Method)**

Run an election for all candidates, then take the top $n$ candidates (often $n = 2$) based on plurality/FPTP. Then run another election between those two.

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This is used to elect the president of France, and also is used in some US cities (and many other places, I'm sure). But it can be time-consuming to run two elections. Is there some way to encode preferences in the vote?

# Runoff elections

**Method 4 (Instant Runoff Method)**

Every vote provides a preference list of their favourite candidates in order. The candidate with the fewest first place votes is eliminated[a], then the votes are tabulated with one less candidate with their votes transferred to their next preference, until there are only two candidates and the one with the most votes wins.

[a] A similar method called *Coombs' rule* instead eliminates the candidate with the most last place votes.

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Used in the House of Representatives in Australia (i.e. decides our prime minister), as well as to determine the president of India and Ireland. It was also used in the *MSS Exec Election 2021*.

# Runoff elections: example

**Definition: Preference profile**

The preference profile of an election is the set of all of the voters' preference lists, with all possible permutations of the candidates recorded in a separate column, and the number of voters for each preference list recorded at the top of each column. For example:

| 5 | 7 | 4 | 3 | 0 | 0 |
|---|---|---|---|---|---|
| A | B | A | C | B | C |
| B | A | C | B | C | A |
| C | C | B | A | A | B |

# Runoff elections: example

Example: Suppose the preference profile of an election is

| 5 | 7 | 4 | 3 | 0 | 0 |
|---|---|---|---|---|---|
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| B | A | C | B | C | A |
| C | C | B | A | A | B |

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**Majority FPTP.** No one wins.

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| C | C | B | A | A | B |

**Majority FPTP.** No one wins.
**Plurality FPTP.** Candidate A wins, as they received 9 first place votes.
**Instant Runoff/Runoff (assuming same preferences).** Candidate C receives 3 first place votes (A receives 9 and B receives 7), so C is deleted and the preferences are transferred so

| 5 | 7 | 4 | 3 |
|---|---|---|---|
| A | B | A | B |
| B | A | B | A |

Therefore B wins the runoff.

# Point scoring systems

**Method 5 (Borda count/Point scoring)**

Allocate preferences, and then give points based on the placings. The candidate with the most points win.

Used in sports more than elections. See: Olympics medal tallies, Brownlow, Dally-M.

# Point scoring systems

**Method 5 (Borda count/Point scoring)**

Allocate preferences, and then give points based on the placings. The candidate with the most points win.

Used in sports more than elections. See: Olympics medal tallies, Brownlow, Dally-M.
Example: Same election as before.

| 5 | 7 | 4 | 3 | 0 | 0 |
|---|---|---|---|---|---|
| A | B | A | C | B | C |
| B | A | C | B | C | A |
| C | C | B | A | A | B |

With a “3,2,1” count where 1st gets 3 points, 2nd gets 2 points, 3rd gets 1 point, the totals are A = 37, B = 36, C = 41. C wins.

# Which system is best?

Three systems, three winners.

- Majority/FPTP: A wins.
- Instant runoff voting: B wins.
- Borda count/point scoring: C wins.

Which system is “best”? Alternatively, which one is “correct”?

# Voting criterion

Mathematicians and political scientists use mathematical **voting criterion** to compare different voting systems. Just a few examples are:

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# Voting criterion

Mathematicians and political scientists use mathematical **voting criterion** to compare different voting systems. Just a few examples are:

- **Majority criterion**: will a candidate always win who is ranked as the unique favourite by a majority of voters?
- **Condorcet criterion**: will a candidate always win who beats every other candidate in pairwise comparison?
- **Independence of irrelevant alternatives**: does the outcome never change if a non-winning candidate is added or removed (assuming unchanged preferences)?

# Voting criterion

- **Monotonicity**: if one or more voters change their ranked preferences by putting one candidate higher, then the overall preference list should either put that candidate higher or be unchanged.

[6] This term comes from economics; an allocation of funds is *Pareto efficient* if there is no other allocation in which some other individual is better off and no individual is worse off. I really don't get the connection.

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- **Monotonicity**: if one or more voters change their ranked preferences by putting one candidate higher, then the overall preference list should either put that candidate higher or be unchanged.
- **Non-imposition**: every possible preference list should be achievable.
- **Pareto efficiency**: if every individual prefers a certain option to another, then the outcome cannot say that the electorate prefers the opposite. IIE, monotonicity and non-imposition imply PE. [6]

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# Voting criterion

| Sort: | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Criterion / Method | Maj-ority | Maj. loser | Mutual maj. | Cond-orcet | Cond. loser | Smith/ ISDA | LIIA | IIA | Clone-proof | Mono-tone | Consis-tency | Partic-ipation | Reversal sym-metry | Polytime/ resolvable | | Summ-able | Later-no- Harm | Later-no- Help | No favorite betrayal | Ballot type | Ranks = | Ranks >2 |
| Approval | Rated [a] | No | No | No [b][c] | No | No[b] | Yes | Yes [d] | Yes[e] | Yes | Yes | Yes | Yes | O(N) | Yes | O(N) | No | Yes [f] | Yes | Appr-ovals | Yes | No |
| Borda count | No | Yes | No | No[b] | Yes | No | No | No | Teams | Yes | Yes | Yes | Yes | O(N) | Yes | O(N) | No | Yes | No | Ran-king | Yes | Yes |
| Bucklin | Yes | Yes | Yes | No | No | No | No | No | No | Yes | No | No | No | O(N) | Yes | O(N) | No | Yes | If equal preferences | Ran-king | Yes | Yes |
| Copeland | Yes | Yes | Yes | Yes | Yes | Yes | No | No [b] | Teams, crowds | Yes | No[b] | No[b] | Yes | $O(N^2)$ | No | $O(N^2)$ | No [b] | No | No[b] | Ran-king | Yes | Yes |
| IRV (AV) | Yes | Yes | Yes | No[b] | Yes | No[b] | No | No | Yes | No | No | No | No | $O(N^2)$ | Yes[g] | O(N!) [h] | Yes | Yes | No | Ran-king | No | Yes |
| Kemeny–Young | Yes | Yes | Yes | Yes | Yes | Yes | Yes | No [b] | Spoil-ers | Yes | No[b] [i] | No[b] | Yes | O(N!) | Yes | $O(N^2)$ [j] | No [b] | No | No[b] | Ran-king | Yes | Yes |
| Highest median/Majority judgment[k] | Rated [l] | Yes [m] | No[n] | No [b][c] | No | No[b] | Yes | Yes [d] | Yes | Yes | No[o] | No[p] | Depends [q] | O(N) | Yes | O(N)[r] | No[s] | Yes | Yes | Scores [t] | Yes | Yes |
| Minimax | Yes | No | No | Yes [u] | No | No | No | No [b] | Spoil-ers | Yes | No[b] | No[b] | No | $O(N^2)$ | Yes | $O(N^2)$ | No [b][u] | No | No[b] | Ran-king | Yes | Yes |
| Plurality/FPTP | Yes | No | No | No[b] | No | No[b] | No | No | Spoil-ers | Yes | Yes | Yes | No | O(N) | Yes | O(N) | N/A [v] | N/A [v] | No | Single mark | N/A | No |
| Score voting | No | No | No | No [b][c] | No | No[b] | Yes | Yes [d] | Yes | Yes | Yes | Yes | Yes | O(N) | Yes | O(N) | No | Yes | Yes | Scores | Yes | Yes |
| Ranked pairs | Yes | Yes | Yes | Yes | Yes | Yes | Yes | No [b] | Yes | Yes | No[b] | No [p][b] | Yes | $O(N^3)$ | Yes | $O(N^2)$ | No [b] | No | No[p][b] | Ran-king | Yes | Yes |
| Runoff voting | Yes | Yes | No | No[b] | Yes | No[b] | No | No | Spoil-ers | No | No | No | No | O(N) [w] | Yes | O(N) [w] | Yes | Yes [x] | No | Single mark | N/A | No [y] |
| Schulze | Yes | Yes | Yes | Yes | Yes | Yes | No | No [b] | Yes | Yes | No[b] | No [p][b] | Yes | $O(N^3)$ | Yes | $O(N^2)$ | No [b] | No | No[p][b] | Ran-king | Yes | Yes |
| STAR voting | No[z] | Yes | No[aa] | No [b][c] | Yes | No[b] | No | No | No | Yes | No | No | Depends [ab] | O(N) | Yes | $O(N^2)$ | No | No | No[ac] | Scores | Yes | Yes |
| Sortition, arbitrary winner[ad] | No | No | No | No[b] | No | No[b] | Yes | Yes | No | Yes | Yes | Yes | Yes | O(1) | No | O(1) | Yes | Yes | Yes | None | N/A | N/A |
| Random ballot [ae] | No | No | No | No[b] | No | No[b] | Yes | Yes | Yes | Yes | Yes | Yes | Yes | O(N) | No | O(N) | Yes | Yes | Yes | Single mark | N/A | No |

**Figure:** Comparison of electoral systems

# Voting criterion

| | Majority | Condorcet | IIA | Mono | NI | Pareto |
|---|---|---|---|---|---|---|
| Plurality/FPTP | Yes | No | No | Yes | Yes | No |
| Runoff | Yes | No | No | No | Yes | No |
| Instant runoff | Yes | No | No | No | Yes | No |
| Borda count | No | No | No | Yes | Yes | No |

**Figure:** Comparison of the voting systems seen prior.

# Arrow's impossibility theorem

We love theorems!

**Theorem (Arrow's impossibility theorem)**

When voters have three or more options, no ranked voting electoral system can convert the ranked preferences of individuals while meeting three of the criteria:

- Pareto efficiency;
- No dictators;
- Independence of irrelevant alternatives.

The practical consequences of the theorem are debatable.[7]

[7] Arrow has said "Most systems are not going to work badly all of the time. All I proved is that all can work badly at times."

# Gibbard–Satterthwaite theorem

**Theorem (Gibbard–Satterthwaite theorem)**

For every electoral method, one of the following must hold:

- There is a dictator;
- The election limits the possible outcomes to two alternatives only;
- The election is susceptible to tactical voting.

In other words, every electoral method is “manipulable”[8] except for two cases: if there is a dictator that decides the vote anyway, or if the rule only has two options.

[8] There exists situations where a sincere ballot does not defend a voter’s preferences best.

# Condorcet's paradox

**Paradox (Condorcet's paradox)**

Collective preferences can be cyclic, even if the preferences of individual voters are not cyclic.

| | 1st | 2nd | 3rd |
|---|---|---|---|
| Voter 1 | A | B | C |
| Voter 2 | B | C | A |
| Voter 3 | C | A | B |

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| Voter 3 | C | A | B |

- If C is chosen as the winner, it could be argued that B should win. B appears above C in two votes, and C appears above B in one. B is preferred to C.
- If B is chosen as the winner... so A is preferred to B. If A is chosen as the winner... so C is preferred to A.
- B is preferred to C, which is preferred to A, which is preferred to B, which is preferred to C...cyclic.

# In summary

In summary, **no election method can meet all of even the most basic criteria one would expect from an election**. The “best” election method is impossible to precisely define, but most can agree it is some combination of:

- Satisfying most of the relevant mathematical voting criteria
- Seeing overall happiness of the population with the results
- Assessing the vulnerability to tactical voting
- and some other things depending on who you ask (is the candidate closest to the average voter?)

There are many more systems used in the real-world...here are just a few.

# Single transferable vote

This is a system commonly used when **multiple winners** are needed, which happens a lot in government (simple example: parliaments where MPs do not have allocated constituencies). Used in the Australian Senate.

**Method 6 (Single transferable vote)**

Allocate preferences. When counting votes, initially allocate to the most-preferred candidate. Candidates are then elected (after reaching some quota) or eliminated, and then surplus votes are transferred.

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The details differ depending on the implementation of the system, but the general concept remains the same. The most common quota formula is the *Droop quota*,

$$\left\lfloor \frac{\text{votes}}{\text{seats} + 1} \right\rfloor + 1.$$

# Party-list proportional representation

Used in most of the rest of the world as another way of doing proportional multiple-winner representation.

**Method 7 (Party-list proportional representation)**

Parties make lists (either open or closed) declaring candidates to be elected. Each voter casts one vote, and seats are distributed to each party proportionate to the number of votes the party receives.

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How to allocate seats when proportions aren't integers? Depends. One common method[9] is the *D'Hondt method*, which computes "quotients" for each party, adding one seat to each party and recalculating until the number of seats is filled. The formula is

$$\text{quotient} = \frac{V}{s+1},$$

where $s$ is the number of allocated seats so far and $V$ is the number of votes received.

[9] There are literally hundreds of these

# Mixed-member proportional representation

Used in the German Bundestag and New Zealand's House of Representatives. My favourite voting system!

**Method 8 (Mixed-member proportional representation)**

Voters cast two votes. One is to decide the representative for a constituency (using FPTP or another plurality/majoritarian system) and another is for a political party (using party-list proportional representation). There may be a minimum threshold needed to be met.

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Note that this isn't parallel voting; the seats are still allocated as if it was one election.

- Allocate seats proportional to parties using a largest remainder method like the D'Hondt method.
- Subtract the number of constituency seats from the party proportional allocation.
- If constituency seats $>$ proportional allocation, add overhang seats (more seats) for the electoral period.

# Schulze method

An example of a Condorcet method, which means that the winning candidate wins every head-to-head matchup.

**Method 9 (Schulze method)**

- Voters specify preferences.
- Construct pairwise preference matrix, and construct a corresponding directed graph.
- Use a graph algorithm such as Floyd–Warshall[a] to identify the *strengths*[b] of the strongest paths.
- From the strengths of the strongest paths, infer the candidate order.

---

[a] All-pairs shortest path on a directed graph with no negative cycles

[b] The strength of a path is the strength of its weakest link, where the strength of path from candidate *A* to *B* is the smallest number of voters in the sequence of comparisons.

# Schulze method

Example: Given the following preference profile:

| 5 | 5 | 8 | 3 | 7 | 2 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| A | A | B | C | C | C | D | E |
| C | D | E | A | A | B | C | B |
| B | E | D | B | E | A | E | A |
| E | C | A | E | B | D | B | D |
| D | B | C | D | D | E | A | C |

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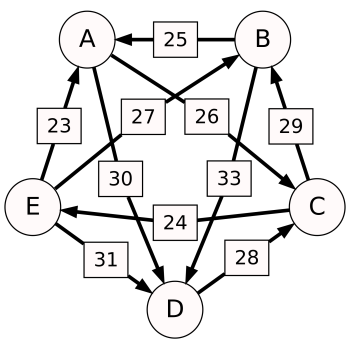
Example: Given the following preference profile:

| 5 | 5 | 8 | 3 | 7 | 2 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| A | A | B | C | C | C | D | E |
| C | D | E | A | A | B | C | B |
| B | E | D | B | E | A | E | A |
| E | C | A | E | B | D | B | D |
| D | B | C | D | D | E | A | C |

**Matrix of pairwise preferences**

| | $d[*, A]$ | $d[*, B]$ | $d[*, C]$ | $d[*, D]$ | $d[*, E]$ |
|---|---|---|---|---|---|
| $d[A, *]$ | | 20 | 26 | 30 | 22 |
| $d[B, *]$ | 25 | | 16 | 33 | 18 |
| $d[C, *]$ | 19 | 29 | | 17 | 24 |
| $d[D, *]$ | 15 | 12 | 28 | | 14 |
| $d[E, *]$ | 23 | 27 | 21 | 31 | |




**Strengths of the strongest paths**

| | $p[*, A]$ | $p[*, B]$ | $p[*, C]$ | $p[*, D]$ | $p[*, E]$ |
|---|---|---|---|---|---|
| $p[A, *]$ | | 28 | 28 | 30 | 24 |
| $p[B, *]$ | 25 | | 28 | 33 | 24 |
| $p[C, *]$ | 25 | 29 | | 29 | 24 |
| $p[D, *]$ | 25 | 28 | 28 | | 24 |
| $p[E, *]$ | 25 | 28 | 28 | 31 | |

# Other systems

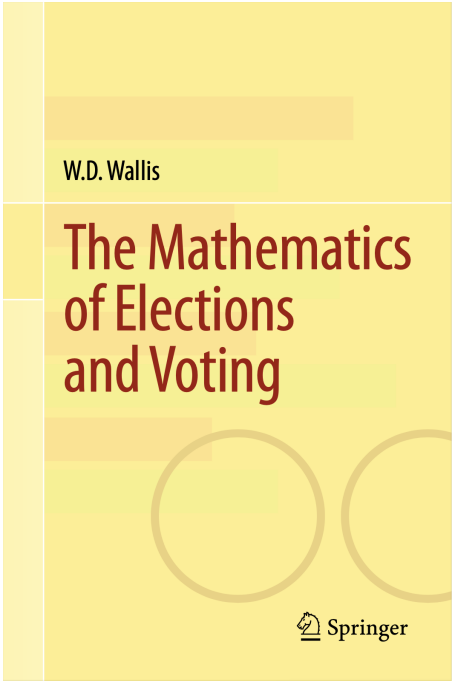
I didn't have time to cover some things I wanted to, such as:

- Electoral colleges (used famously to decide the US presidency)
- Cumulative voting (you're given $n$ votes, distribute them however you want)
- Approval voting (approve a number of candidates, winner is most approved)
- Majority bonus system (extra seats for the best!)
- Random ballots

...but maybe that is a good thing.

# Recommended reading

- This book covers most of the first half of this talk:




- Wikipedia. Seriously, it was invaluable for this talk. Wikipedia has a ridiculous amount of information on specifically voting systems for some reason.

# Thank you!

Thanks for coming :)