Quantum Theory in a Shoe String

MSS Talk

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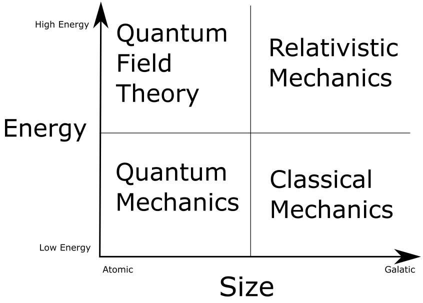
University of Queensland

2021 Mar 05

Not a physicist!

So take what I say with a jug of salt...

Overview



High Energy
Quantum Field Theory
Relativistic Mechanics
Energy
Quantum Mechanics
Classical Mechanics
Low Energy
Atomic
Galatic
Size

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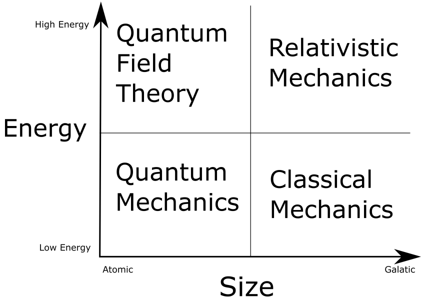

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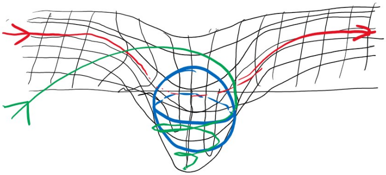

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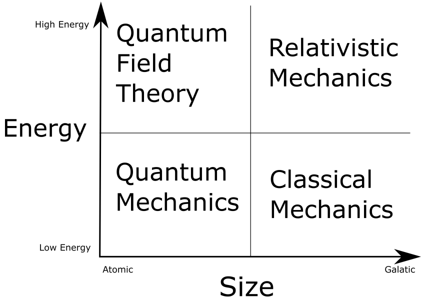

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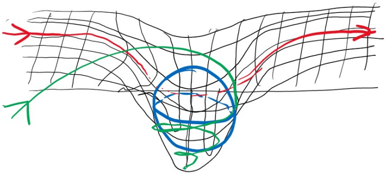

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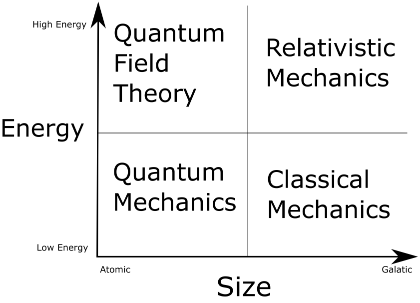

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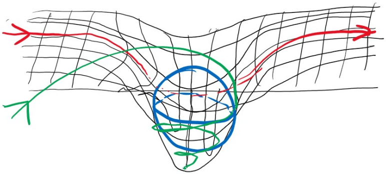

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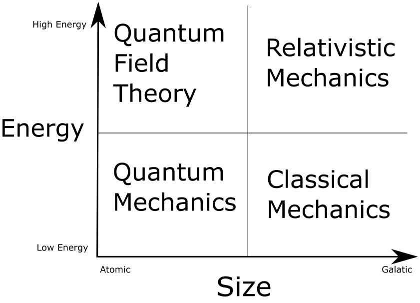

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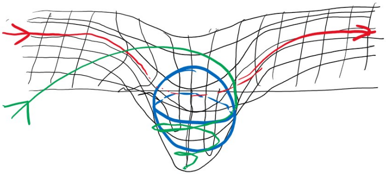

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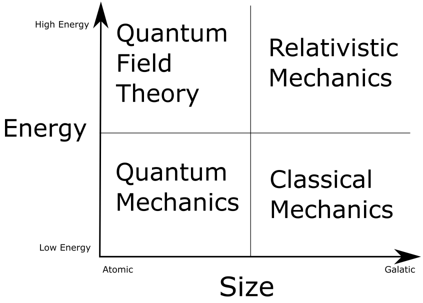

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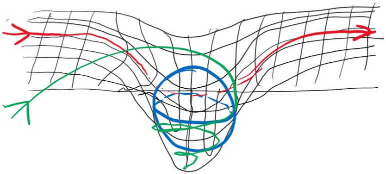

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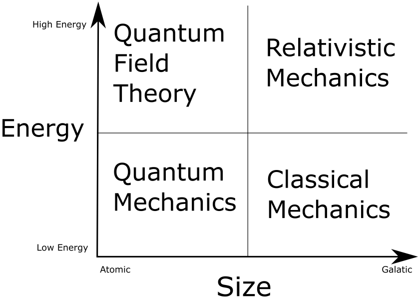

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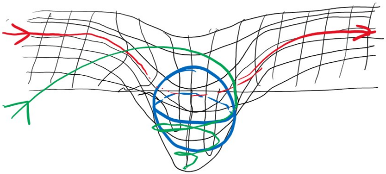

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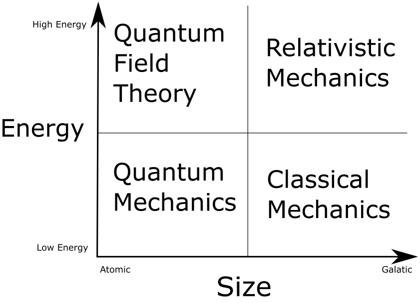

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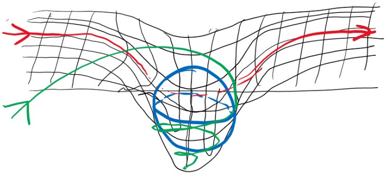

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Today:

1. What are Strings?
2. Quantised Strings
3. Compactifications

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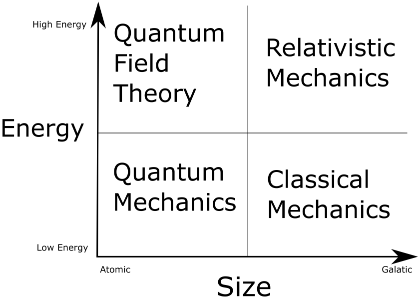

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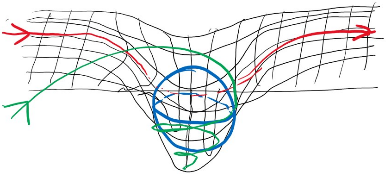

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- $(M^{1;25}; \;)$ 26 dimensional Minkowski space
- $X : M^2 \mapsto M^{1;25}$ an embedding

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Note*: a moving particle has 6 "dimensions" (3-spatial, like the above 26 dimension)

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Quantum String

General Idea:

Classical Physics =) Quantum Physics

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Results:

T is *really* small

Discrete quantity (half integer spin) from winding number

4 ‘observable’ dimensions

Gravitons! (closed string)

Translation invariant in only 4 dimensions - the others can do whatever