Quantum Theory in a Shoe String

MSS Talk

Lawrence Lo

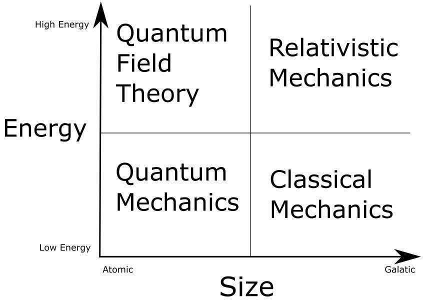
School of Mathematics and Physics
University of Queensland

2021 Mar 05

Not a physicist!

So take what I say with a jug of salt...

Overview


High Energy
Quantum Field Theory
Relativistic Mechanics
Energy
Quantum Mechanics
Classical Mechanics
Low Energy
Atomic
Galatic
Size

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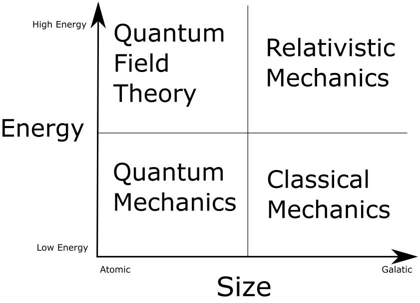

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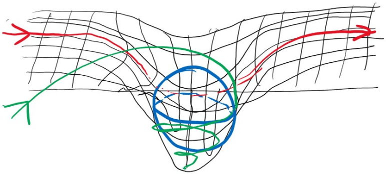

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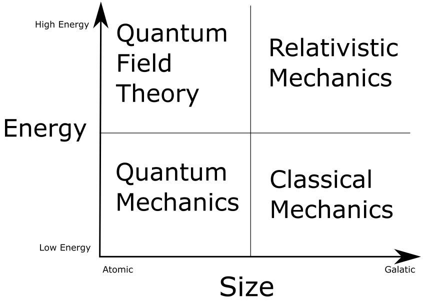

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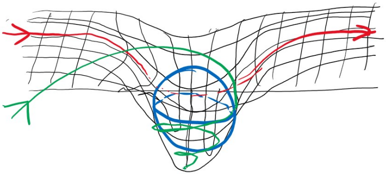

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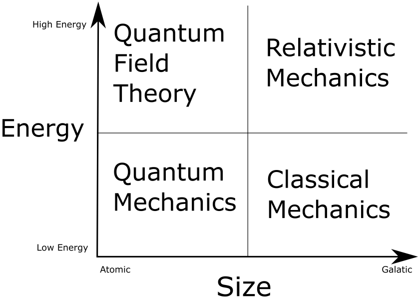

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- Gravity in Relativity: Curvature in Spacetime

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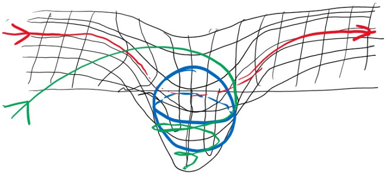

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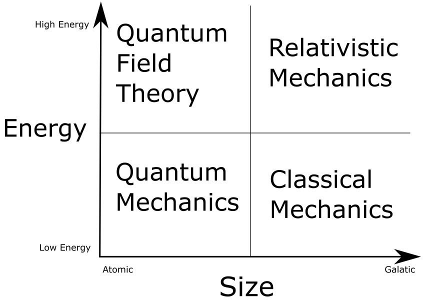

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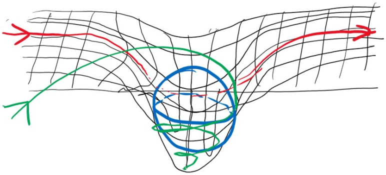

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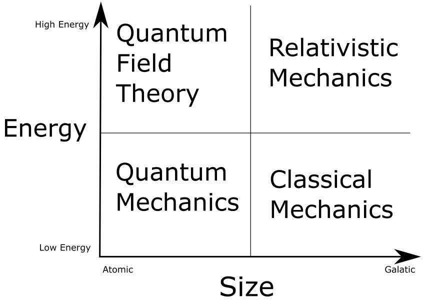

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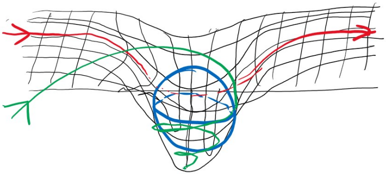

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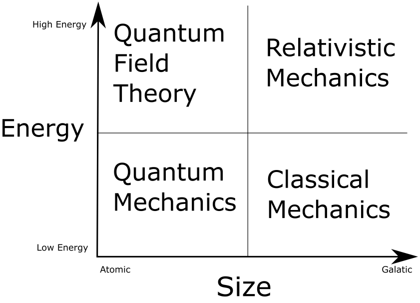

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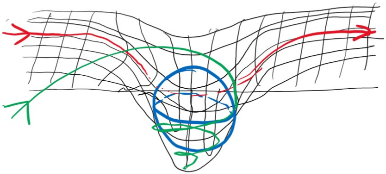

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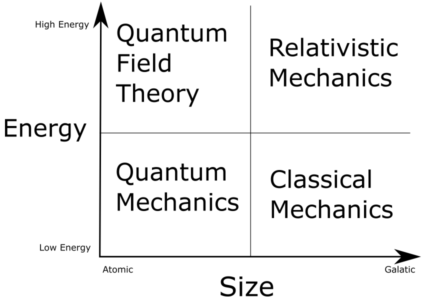

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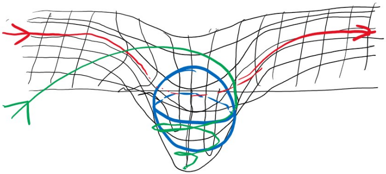

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3. Compactifications

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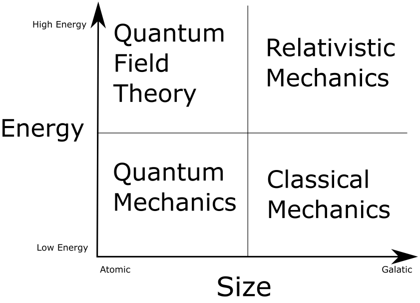

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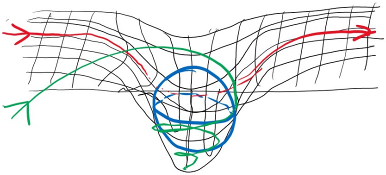

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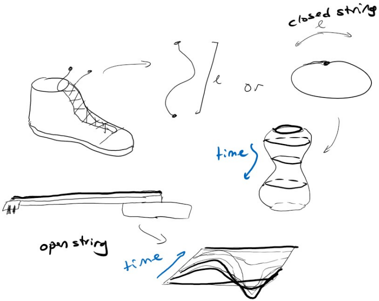

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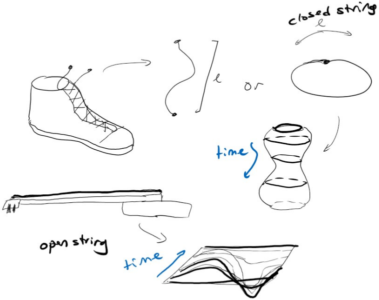

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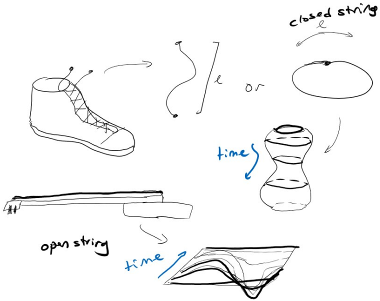

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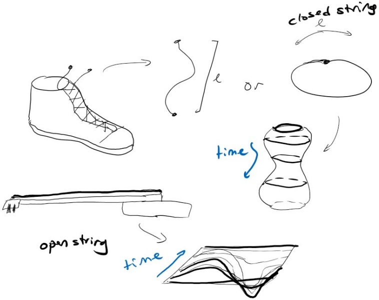

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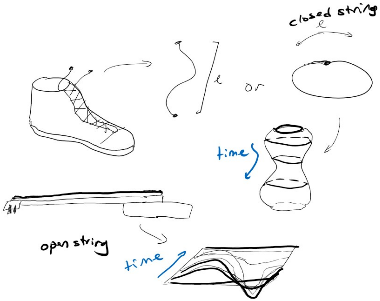

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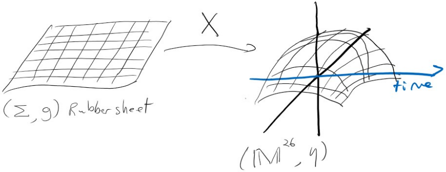

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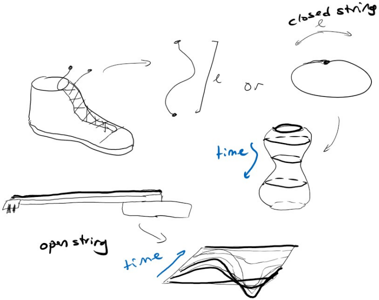

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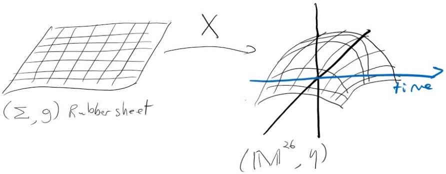
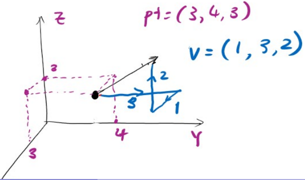


Note*: a moving particle has 6 "dimensions" (3-spatial, like the above 26 dimension)



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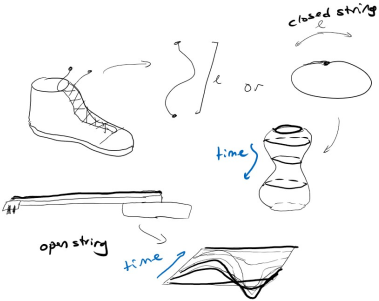

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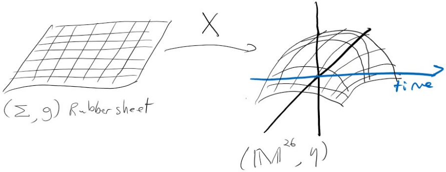

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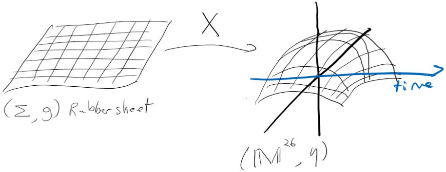

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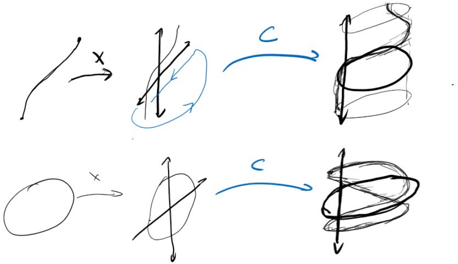

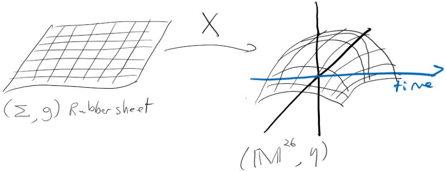

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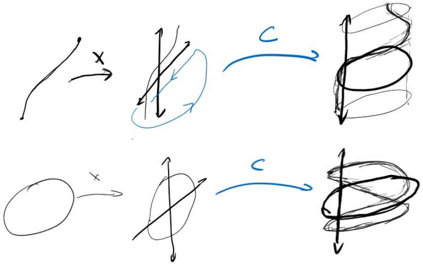

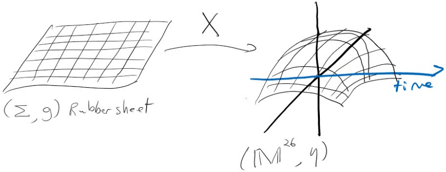

Data:

- (M^2, g) 2-Riemannian manifold
- $(M^{1,25}, \eta)$ 26 dimensional Minkowski space
- $X : M^2 \mapsto M^{1,25}$ an embedding
- $C : M^{1,25} \to M^{1,3} \times T$

Results:

- T is *really* small
- Discrete quantity (half integer spin) from winding number
- 4 'observable' dimensions
- Gravitons! (closed string)
- Translation invariant in only 4 dimensions - the others can do whatever